Abstract – This research proposes a new magnetic resonant WPT (wireless power transfer) system which transfers the power to multiple receivers. In general, the received power at each receiver differs by the distance between the transfer and receiver coils. However, this controls the received power by varying the resistance of receivers. Optimum shape of the transfer coil and size of receiver coils are also considered.

I. Introduction

By the impact of the first WPT by the magnetic resonant coupling proposed by MIT in 2007, many researches and developments have been following in this field. Methods of using electromagnetic induction and radio wave transmitting have been actively conducted in recent years. Most of conventional studies of WPT have concentrated on one receiver. There are much more difficulties for applying directly to multi receivers. However, supplying power to multiple receivers is expected in many cases, for example, IoT, mobile devices, and factory automations. Moghadam[1] proposed that there is a problem which receivers close to a transfer coil receive a lot of power. Receivers placed far from the transfer coil receive a little of power. Fu[2] notifies the phenomena of the cross coupling in multiple-receiver wireless power transfer systems. Although there are some basic contributions reported in these literature, the key deficiency of the recent research work in this area is the complete omission or use of highly simplistic models of the coupling effects between transfer and receiver coils.

Therefore, the purpose of this study is to consider and construct efficient power supply system between transfer coil and receiver coils. We made an environment in which the cross coupling between the transfer and receiver coil can be ignored by examining the shape of transfer coil. There by that can control a received power by changing the resistance of receivers. It has robustness in a limited space when employing magnetic resonant coupling for multiple receivers. It can be applied to electronic devices with low power consumption such as IoT sensors.

II. Preliminary

A. Basic Characteristics of WPT for multiple receivers

First, we derived the basic circuit characteristics of WPT for multiple receivers, as shown in Fig. 1, by solving the Z-matrix. The relationship of current and voltage is given by following equation. This is obtained when the distance between receivers is sufficiently far away and the influence due to the cross coupling can be ignored.

\[
\begin{bmatrix}
V_1 \\
0 \\
\vdots \\
0 \\
0 \\
\end{bmatrix} =
\begin{bmatrix}
 r_1 & j \omega M_{t1} & j \omega M_{t2} & \cdots & j \omega M_{tn} \\
 j \omega M_{t1} & r_1 + R_{e1} & 0 & \cdots & 0 \\
 j \omega M_{t2} & 0 & r_2 + R_{e2} & \cdots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 j \omega M_{tn} & 0 & 0 & \cdots & r_n + R_{en}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{n-1} \\
I_n \\
\end{bmatrix}
\]

(1)

Here, \( I_n \) is the current, \( V_n \) is the voltage, \( R_{en} \) is the resistance of receiver, \( r_n \) is the parasitic capacitance of receiver coil and \( M_{en} \) is the mutual inductance between transfer and receiver coil.

The transfer power \( P_{in} \), the received power \( P_n \), and the efficiency of the whole system \( \eta \) are calculated by following equations.

\[
\begin{align*}
P_{in} &= Re(V t) \\
P_n &= Re(V t - I t)
\end{align*}
\]

(2)
\begin{align}
\eta_1 &= \frac{P_1}{P_{in}} \\
\vdots &= \eta = \eta_1 + \cdots + \eta_n = \sum_{i=1}^{n} \eta_i \\
\eta_n &= \frac{P_n}{P_{in}}
\end{align}

(3)

Next, following equations are obtained in case that each mutual inductance \( M_{tn} \) can be regarded as a fixed value and the conditions of receivers are same.

\begin{align}
I_t &= \frac{r+R}{r_t R+(n-1)\omega^2 M^2 + r_t r} V_t \\
I_n &= \frac{-j\omega M}{r_t R+(n-1)\omega^2 M^2 + r_t r} V_t \\
V_1 &= \text{constant} \\
V_n &= \frac{r+R}{r_t R+(n-1)\omega^2 M^2 + r_t r} V_t \\
P_t &= \frac{r+R}{r_t R+(n-1)\omega^2 M^2 + r_t r} V_t^2 \\
P_n &= \frac{R(\omega M)^2}{(r_t R+(n-1)\omega^2 M^2 + r_t r)^2} V_t^2
\end{align}

(4)

(5)

(6)

In addition, the following equation (7) is obtained by rearranging equation (3).

\[ \eta = \frac{nR\omega^2 M^2}{(r+R)(r_t R+n\omega^2 M^2 + r_t r)^2} \]

(7)

**B. Considering transfer and receiver coil**

We considered the condition that the mutual inductance \( M_{tn} \) can be regarded as a fixed value in order to apply the equation (6). In general, the mutual inductance between two coils is expressed in the following equation.

\[ V_{12} = -N_1 \frac{\Delta \phi_{1+1}}{\Delta t} = -M_{12} \frac{\Delta I_{2}}{\Delta t} \]

(8)

From this equation of electromotive force, the mutual inductance between two coils does not change in a state of constant magnetic flux density. Therefore, we used a coil generating a constant magnetic field as a transfer coil, which is called Helmholtz coil as shown in Fig. 2.

A Helmholtz coil is a pair of coils of same radius which are placed by intervals of the radius. It generates a uniform magnetic field between two coils. The magnetic flux density at the midpoint \( O \) on the central axis between two coils is applied to the following equation by the law of Bio-Savart. \( B \) is the magnetic flux density, and \( \mu_0 \) is the vacuum permeability.

\[ B = \mu_0 l a^2 \left( \frac{1}{2(a^2+(d+x)^2)^2} + \frac{1}{2(a^2+(d-x)^2)^2} \right) \]

(9)

Then, we considered a size of receiver coils. The condition expressed by the equation (4) is established only when the mutual inductance between two receiver coils is sufficiently little. Thus, we considered that the cross coupling between two receiver coils may be occurred following Table I.

In general, the equation for obtaining the mutual inductance between two circular coils is called Neumann’s formula [4] and is expressed by the equation (10). \( D \) is the distance between \( dl_1 \) and \( dl_2 \), and \( \oint dl \) is the line integral.

\[ M = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{dl_1 dl_2}{D} \]

(10)

Moreover, the following equation (11) is obtained by rearranging equation (10) in case that those circular coils are one-loop coils, and the relationship at this time are given in Fig. 3.

\[ M = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{r^2 \cos(\theta_1 - \theta_2)}{\sqrt{r^2 + g^2 - 2r^2 \cos(\theta_1 - \theta_2)}} d\theta_1 d\theta_2 \]

(11)

We studied the size of the receiver coil considering the size of the whole experiment environment based on the equation. The calculation results of the relationship between the air gap and mutual inductance when the diameter of two coils is 100[mm] on the basis of the equation (11) is shown in Fig. 4. By this results, two receiver coils should separate by 60[mm] at least, in order to the mutual inductance is ignored.

From the above, the conditions of the transfer and receiver coil are set as shown in Table I.
The result of simulating the magnetic field generated by the transfer coil is shown in Fig. 5 which is simulated by Femtet that is a software that can analyze the magnetic field by the finite element method. The magnetic field spreads uniformly between the two coils, and its magnetic flux density is about 450[µT].

The scheme of the whole system is shown in Fig. 6. The efficiency of transmitting power will be low unless we produce a resonance circuit considering the resonance conditions of transfer and receiver.

Thus, we measured the inductance of the transfer and receiver coil by using a LCR meter, and the capacitance required to resonate using the capacitance was derived based on the equation (12). $f$ shows the resonance frequency, which is 75[kHz] in our study. In addition, the specification of the experimental circuit is shown in Table II and the whole experimental environment is shown in Fig. 7.

$$ f = \frac{1}{2\pi \sqrt{L_C C_T}} = \frac{1}{2\pi \sqrt{L_n C_n}} \quad (12) $$
Table II. Specification of the experimental circuit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmit Power</td>
<td>0.163 [W]</td>
</tr>
<tr>
<td>Resonance Frequency</td>
<td>75 [kHz]</td>
</tr>
<tr>
<td>Resonance Capacitor ($C_t$)</td>
<td>690 [pF]</td>
</tr>
<tr>
<td>Transmitter Coil ($L_t$)</td>
<td>6.36 [mH]</td>
</tr>
<tr>
<td>Resonance Capacitor ($C_r$)</td>
<td>0.44 [μF]</td>
</tr>
<tr>
<td>Receiver Coil ($L_r$)</td>
<td>10.2 [μH]</td>
</tr>
</tbody>
</table>

III. Experimental Results

A. In the case: One receiver coil

First, we describe a case that there is only one receiver coil. The settings of the position at the measurement is shown in Fig. 8.

The measurement results of the transfer efficiency when we vary the load resistance of receivers are shown in Fig. 9.

The maximum transfer efficiency was about 14.7 [%] at 5[Ω]. The reason why the efficiency decreases is considered to be due to the large difference between the size of the transfer coil and receiver coils, and much leakage magnetic flux is generated.

Furthermore, the maximum efficiency is often calculated by $kQ$ using the $k$ means coupling coefficient and the $Q$ value that is representing the performance of the coil in order to realize the maximum efficiency for WPT. The coupling coefficient $k$, $Q$ value and the transfer efficiency $\eta_{\text{max}}$ are expressed by the following equations.

\[
k = \frac{M}{\sqrt{L_t L_r}} \quad (13)
\]

\[
Q_t = \frac{\omega L_t}{\sqrt{R_t}}, \quad Q_r = \frac{\omega L_r}{\sqrt{R_r}} \quad (14)
\]

\[
k^2 Q_t Q_r = \frac{\omega M}{\sqrt{R_t R_r}} \quad (15)
\]

\[
\eta_{\text{max}} = \frac{k^2 Q_t Q_r}{(1 + k^2 Q_t Q_r)^2} \quad (16)
\]

In addition, the relationship between the value of $kQ$ and the transfer efficiency is expressed as shown in Fig. 10.

In general, the coupling coefficient $k$ depends on the distance to the transfer coil, and the efficiency varies greatly due to the mutual inductance. However, the mutual inductance can be regarded as constant in our research. Therefore, our system is considered to be a system resistant to change in position.

In order to improve the efficiency without changing the size of coils under the above conditions, increasing the number of turns of coils to increase the inductance, increasing the resonant frequency, or using a larger diameter of coil that the application can reduce the loss resistance.
B. In the case: Two receiver coils

Secondly, we describe a case that there are two receivers. The settings of the position at the measurement is shown in Fig. 11.

![Fig. 11. Position for two receiver coils](image)

The results of the ratio of received power by two receivers when varying the load resistance and the overall efficiency of whole system obtained by equation (7) are as shown in the following Table III. From that results, the maximum error from the calculation obtained by equation (6) was about 4.4\%], and almost the measurement theory was obtained. The overall efficiency is lower than in the case: one receiver coil. This is considered to be due to the fact that the value of the load resistance of the receivers is different from the condition for maximizing the efficiency.

<table>
<thead>
<tr>
<th>Ratio of Resistance (Receiver(1 : 2))</th>
<th>Ratio of Received Power (Calculation)</th>
<th>Ratio of Received Power (Experiment)</th>
<th>Maximum Difference</th>
<th>Overall Efficiency (Calculation)</th>
<th>Overall Efficiency (Experiment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 1</td>
<td>50.0 : 50.0</td>
<td>49.2 : 50.8</td>
<td>0.8%</td>
<td>14.1%</td>
<td>11.4%</td>
</tr>
<tr>
<td>1 : 2</td>
<td>66.7 : 33.3</td>
<td>62.7 : 37.3</td>
<td>4.0%</td>
<td>15.3%</td>
<td>9.4%</td>
</tr>
<tr>
<td>1 : 3</td>
<td>75.0 : 25.0</td>
<td>70.6 : 29.4</td>
<td>4.4%</td>
<td>15.2%</td>
<td>9.3%</td>
</tr>
<tr>
<td>2 : 1</td>
<td>33.3 : 66.7</td>
<td>31.1 : 68.9</td>
<td>2.2%</td>
<td>15.7%</td>
<td>10.3%</td>
</tr>
<tr>
<td>2 : 3</td>
<td>60.0 : 40.0</td>
<td>61.1 : 38.8</td>
<td>1.2%</td>
<td>17.2%</td>
<td>10.9%</td>
</tr>
</tbody>
</table>

Table IV. Results of three receivers

<table>
<thead>
<tr>
<th>Ratio of Resistance (Receiver(1 : 2 : 3))</th>
<th>Ratio of Received Power (Calculation)</th>
<th>Ratio of Received Power (Experiment)</th>
<th>Maximum Difference</th>
<th>Overall Efficiency (Calculation)</th>
<th>Overall Efficiency (Experiment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 1 : 1</td>
<td>33.3 : 33.3 : 33.3</td>
<td>35.7 : 31.6 : 32.7</td>
<td>2.4%</td>
<td>15.0%</td>
<td>12.1%</td>
</tr>
<tr>
<td>1 : 2 : 1</td>
<td>40.0 : 20.0 : 40.0</td>
<td>42.1 : 20.5 : 37.4</td>
<td>2.6%</td>
<td>13.0%</td>
<td>11.6%</td>
</tr>
<tr>
<td>1 : 3 : 2</td>
<td>54.5 : 18.2 : 27.3</td>
<td>49.5 : 20.9 : 29.6</td>
<td>5.0%</td>
<td>10.3%</td>
<td>8.5%</td>
</tr>
<tr>
<td>2 : 3 : 3</td>
<td>42.9 : 28.6 : 28.6</td>
<td>39.8 : 28.1 : 32.1</td>
<td>3.5%</td>
<td>7.5%</td>
<td>10.3%</td>
</tr>
<tr>
<td>3 : 1 : 3</td>
<td>20.0 : 60.0 : 20.0</td>
<td>21.1 : 54.8 : 24.1</td>
<td>5.2%</td>
<td>9.5%</td>
<td>8.3%</td>
</tr>
</tbody>
</table>
The overall efficiency is low because the difference in size between the transfer and receiver coil, and the coupling coefficient becomes small.

Furthermore, we examined the efficiency of the whole system when increasing the number of receivers. From the equation (7), the relationship between the efficiency, the number of receivers and the resistance of receivers is expressed as shown in Fig. 13 in the case of our study.

![Fig. 13. Efficiency of the whole system](image)

From this result, even if the resistance of receivers is not the optimal condition, the efficiency of the whole system improves by increasing the number of receivers. It is thought that the efficiency of the whole system will be higher when this system is implemented on a large scale.

IV. Conclusion

In this paper, we have examined a one-to-multiple WPT using magnetic resonant coupling. We considered deriving the characteristic equation of the circuit and using the Helmholtz coil as the transfer coil.

We have confirmed that the ratio of the received power can be controlled and optimized by the value of the resistance of each receivers. The problem is that the overall efficiency is low due to the difference in size between the transfer and receiver coil. Moreover, the restriction on the distance between receivers is large. However, these problems can be solved by implementing a larger system. We think that this system suitable for batch power supply to a large number of receivers in a space where people do not enter.

For future work, we plan to add a DC/DC converter controlled by a microcomputer to charge the battery of the receiver. Also, we aim to construct a system that it has robustness in a limited space and enables more efficient and fair WPT for multiple receivers.

References


