Optimal design of allpass digital filters using artificial bee colony

Wei-Der Chang[1*], Shing-Tai Pan[2], Kuo-Hua Cheng[1], Ming-Chieh Hsu[1]

[1]Department of Computer and Communication, Shu-Te University, Kaohsiung, Taiwan
[2]Department of Computer Science and Information Engineering, National University of Kaohsiung, Kaohsiung, Taiwan
* Corresponding author: Tel: +886-7-6158000 #4808; Fax: +886-7-6158000#4899
E-mail: wdchang@stu.edu.tw

Abstract - This paper applies a novel artificial bee colony algorithm to solve the design problem of allpass digital filters. We wish that the phase response of allpass filter can meet the desired specification. To achieve this aim, the ABC algorithm is utilized to update the related filter coefficients such that certain cost function of the algorithm can be minimized as possible as much. Finally, numerical simulation results will demonstrate the feasibility and effectiveness of the proposed scheme.

I. Introduction

Karaboga and his research team initially presented a new evolutionary algorithm called artificial bee colony (ABC) in recent years [1]-[3]. This method is also a population-based algorithm like the other swarm intelligences. Basically, the concept of the ABC algorithm is stimulated from the social behavior of honey bee swarm. In the ABC algorithm, the colony of artificial bees is divided into three kinds of bees such as employed bees, onlookers, and scouts [1]. Each colony has its individual function and role. By means of these bee colony manipulations, the system optimization design can be achieved. In [4], the authors used the ABC algorithm to model the IIR digital filter and the simulation results showed a good performance. In [5], the ABC algorithm was employed to search out the optimal combinations of different operating parameters for three used non-traditional machining (NTM) processes. The proposed method has a superior performance over other traditional schemes. Another central subject of this study is concerned with the allpass digital filter. As its name disclosure, such filter has the magnitude response of constant value for all of frequencies, i.e., independently of frequency. To offer a phase modification without changing the signal magnitude is the main utility of the allpass filter. This kind of digital filter has usually been applied in the signal processing areas such as multirate filtering, group-delay equalization, complementary filter banks, and other applications [6]-[9].

This paper will attempt to utilize the ABC optimal algorithm to solve the design problem of the allpass digital filter. Filter parameters are adjusted via the ABC algorithm such that the derived phase response approaches a previously desired one. The remainder of this study is organized as follows. Section II addresses the structure of the allpass digital filter and its phase responses. In Section III, we will proposes the whole design steps for the phase design of the allpass filter by using the ABC algorithm. Some simulations and examinations are given in Section IV to verify the efficiency of the proposed method. Finally, a brief conclusion is stated in Section V.

II. Phase response of an allpass digital filter

The following is the standard transfer function of an N-th order IIR allpass filter with real coefficients:

$$H(z)=\frac{a_N+a_{N-1}z^{-1}+\cdots+z^{-N}}{1+a_1z^{-1}+\cdots+a_Nz^{-N}}=\frac{z^{-N}D(z^{-1})}{D(z)} \tag{1}$$

where a_i is the filter coefficient and $D(z)=\sum_{i=0}^{N}a_iz^{-i}$ with $a_0=1$. Let $z=e^{j\Omega}$ to further obtain the filter's frequency response, where Ω represents the digital frequency. Hence the magnitude response of Eq. (1) is given by

$$|H(\Omega)|=\left|\frac{1+a_1e^{j\Omega}+\cdots+a_Ne^{jN\Omega}}{1+a_1e^{-j\Omega}+\cdots+a_Ne^{-jN\Omega}}\right|=1 \tag{2}$$

It is easily seen that it is equal to one at all digital frequencies and is independent of the filter coefficients as well. By means of simple calculation, on the other hand, the phase response can be expressed as

$$\theta(\Omega)=-N\Omega+2\tan^{-1}\left(\frac{\sum_{i=1}^{N}a_i\sin(i\Omega)}{\sum_{i=1}^{N}a_i\cos(i\Omega)}\right). \tag{3}$$

Let vector $A\equiv[a_1,a_2,\cdots,a_N]$ be a collection of all designed filter coefficients and this parameter vector entirely dominates the phase response of the allpass filter. This vector is called the food source in the viewpoint of ABC algorithm. Because Eq. (3) contains a highly nonlinear function $\tan^{-1}(\cdot)$, it should be modified as another mathematical expression to further get the solution in the traditional method. In this study, however, we can proceed to deal with the phase design of the allpass filter by using Eq. (3) directly.

III. Phase response design using ABC algorithm

The artificial bee colony (ABC) algorithm was presented by Karaboga and his research team and has been shown to be a powerful search method for solving optimization problem. The central idea of the ABC algorithm is motivated from the biological behaviour of bee colony. Artificial bees can be divided into three different types including employed bees, onlookers, and scouts. The employed bee is defined as a bee that is going to the food source previously visited by itself, and a bee which is waiting on the dance area for making decision to choose a food source is called the onlooker, and finally the scout bee means that a bee is carrying out random search for food sources. First half of the colony is composed of employed artificial bees and the remainder half then contains onlooker bees. Every employed bee corresponds to one food source and this means that the number of employed bees is equal to the number of food source. Moreover, an employed bee will become a scout when its food source is exhausted by itself and onlooker bees [1].

In the ABC algorithm, each food source represents a candidate solution of the optimized problem and here we let $A_i = [a_{i1}, a_{i2}, \cdots, a_{iN}]$ denote the *i*th food source position (i.e., the designed filter coefficients vector) with *N* elements. Note that the number of the employed bees or the onlooker bees is equal to the number of the candidate solutions in the given optimization problem. Furthermore, each food source is required to calculate the corresponding cost function (amount of nectar) to evaluate its performance. Here we simply define the cost function as

$$CF = \int_{\Omega_{\min}}^{\Omega_{\max}} \left|\theta_d(\Omega) - \theta(\Omega)\right| d\Omega \qquad (4)$$

where θ_d is the desired phase response given by the designer, θ is the actual phase response of the allpass filter as described by Eq. (3), $\Omega_{\min}$ and $\Omega_{\max}$ are the lower and upper bounds of the digital frequency, respectively. According to the cost function, a probability $P(A_i)$ associated with the food source A_i need to be calculated and this value determines whether an onlooker bee chooses a new food source. It is defined by

$$P(A_i) = \frac{CF(A_i)}{\sum_{n=1}^{H} CF(A_n)}, \text{ for } i = 1, 2, \cdots, H \qquad (5)$$

where $CF(A_i)$ represents the cost function obtained by A_i and H denotes the number of all food sources, i.e., population size. It is clear from Eq. (5) that the probability is proportional to the nectar amount of the food source. Besides, the food source position of a new candidate is produced by the following equation:

$$a_{ij}^{new} \leftarrow a_{ij} + \phi_{ij}(a_{ij} - a_{kj}), \quad i = 1, 2, \cdots, H, \quad j = 1, 2, \cdots, N \qquad (6)$$

where a_{ij}^{new} is the new candidate position, ϕ_{ij} is a uniformly random number chosen from certain interval, and k is a random integer from $\{1, 2, \cdots, H\}$ but $k \neq i$. The ABC algorithm principally uses Eq. (6) to achieve the optimization purpose.

Design steps of the phase response of the allpass digital filter by using the ABC algorithm can be summarized below.

I. Randomly produce an initial population with H food sources.

II. If the assigned number of generations G is achieved, then terminate the algorithm.

III. Calculate every objective function $CF(A_i)$ and every probability $P(A_i)$, respectively .

IV. Create a random number *pr* from the interval $[0, 1]$. If $P(A_i) \leq pr$, then a candidate food source A_i^{new} is generated according to Eq. (6).

V. Calculate the cost function $CF(A_i^{new})$ and compare it with $CF(A_i)$. If $CF(A_i^{new}) < CF(A_i)$, then let A_i^{new} take the place of A_i; otherwise, the old food source A_i remains.

VI. If the number of generations is equal to the value, *limit*, then check whether the positions of every A_i have been changed. If no any modification, this food source is discarded and then reproduce a new food source at random.

VII. Go back to Step II.

IV. Simulation results

In this section, we will demonstrate the feasibility of our proposed method by the following simulation example. The related variables used in the ABC algorithm are given by population size $H = 20$, number of iterations $G = 1000$, and $limit = 40$. The perturbing random number ϕ_{ij} in Eq. (6) is generated from the interval $[-0.1, 0.1]$ uniformly. Our purpose here is to design an allpass digital filter to approximate a Hilbert transformer via the proposed ABC algorithm. The phase response of a Hilbert transformer is described by [8]

$$\theta_d(\Omega) = -N\Omega - \frac{\pi}{2}, \quad \Omega_{\min} \leq \Omega \leq \Omega_{\max} \qquad (7)$$

where $\Omega_{\min} = 0.06\pi$ and $\Omega_{\max} = 0.94\pi$ are chosen and the order of the allpass filter is set to $N = 10$. After performing the proposed ABC algorithm, the derived results are shown in Figs. 1 and 2, and listed in TABLE I, respectively. Tab.1 lists the designed filter coefficients and their corresponding cost functions according to various initial conditions Run 1 ~ Run 5. All of maximum radiuses of poles are less than one and this means that the derived digital filters are stable. Fig. 1 is then the comparison of phase responses between the desired and the actual ones. A satisfactory approximation can be obtained. In addition, the trajectories of the cost functions with respect to the generation are displayed in Fig. 2.

filter.

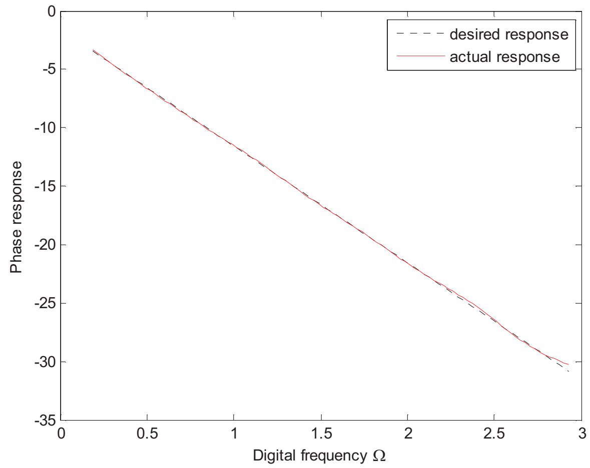

Fig. 1. Phase response for Run 3.

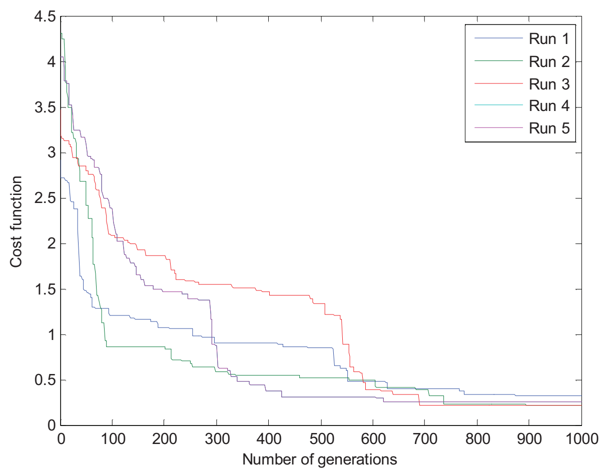

Fig. 2. Trajectories of cost functions for Run 1 ~ Run 5.

V. Conclusions

In this paper, we have presented a simple design method for the phase response of the allpass digital filter. A novel evolutionary computation named the artificial bee colony (ABC) that mimics the social behavior of honey bees is employed to optimally update the filter coefficients. From simulation results, we conclude that the proposed method can do very well on the design problem of the allpass digital filter.

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TABLE I. The derived filter coefficients and the corresponding cost functions for Run 1 ~ Run 5.

	Run 1	Run 2	Run 3	Run 4	Run 5
a_1	-0.9460	-0.9916	-0.9864	-0.9814	-0.9710
a_2	0.3813	0.4708	0.4501	0.4541	0.4428
a_3	-0.3363	-0.4260	-0.4195	-0.4114	-0.4174
a_4	0.2376	0.2963	0.2832	0.2192	0.2447
a_5	-0.2859	-0.2837	-0.2744	-0.1889	-0.2106
a_6	0.2503	0.2186	0.2252	0.1259	0.1202
a_7	-0.2466	-0.2065	-0.1785	-0.0811	-0.0668
a_8	0.1742	0.1179	0.1254	0.0454	0.0406
a_9	-0.0964	-0.0860	-0.0826	-0.0406	0.0139
a_{10}	0.0362	0.0388	0.0212	0.0023	-0.0185
Cost function	0.3223	0.2225	0.2135	0.2507	0.2913
Maximum radius of pole	0.8637	0.8948	0.8747	0.9028	0.8428