Polygon Fracturing Method Considering Maximum Size Limit

Taiki Matsuzaki, Kunihiro Fujiyoshi, Tomohiko Hotta

Department of Electrical and Electronic Engineering
Tokyo University of Agriculture and Technology
2-24-16 Nakacho, Koganei, Tokyo, 1848588 Japan
s226355x@st.go.tuat.ac.jp, fujiyosi@cc.tuat.ac.jp

Abstract— Variable shaped-beam electron beam lithography systems are widely used for mask writing. The exposure data, which is an input for variable shaped-beam mask writing, must be a set of rectangles with considering maximum size limit. It is also crucial to fracture the layout into as few rectangles as possible for reducing the number of times the beam irradiated. Although several methods have been proposed, there is still no method to obtain an optimal solution in practical calculation time with maximum size limit. In this paper, we propose four types of *Optimal Single-Partitions* and prove these partitions guarantee that the optimal solutions are not lost. We performed computational experiments to evaluate the performance of methods based on these *Optimal Single-Partitions*. Our methods yield better solutions than the previous method and are faster than ILP in many cases.

I. INTRODUCTION

Large-scale integrated circuits are mass-produced by using a transcription device called a stepper, which transfer circuit patterns onto wafers at a reduced size. Variable shaped-beam(VSB) electron beam lithography systems are widely used to write mask based on arbitrary circuit patterns on photomasks[1], which are the original plates to be transferred. VSB mask writing machines have high resolution and can draw at high speed, but there are some restrictions. First, the exposure unit is a rectangle. All polygons that compose the layout must be fractured into a set of rectangles. Secondly, there is the size limit of the rectangle that the machine can write. All rectangles must be less than or equal to maximum size limit. Addly, it is important to fracture the layout into as few rectangles as possible. Since each rectangle is written one by one by the exposure system, a larger number of rectangles increases the number of electron beam irradiations and also increases the photomask manufacturing cost[2].

Some methods were proposed to fracture an input rectilinear polygon into rectangles not less than the size limit. Kahng et al.[3] proposed ILP (integer linear programming) based method, considering suppression of generation of small-width rectangles and maximum size limit. ILP may obtain one of the optimal solutions, but requires super-polynomial calculation time, so the solution cannot be obtained in practical calculation time in many cases. Therefore, the method of Kahng et al. fractures the input layout into polygons with less than a certain number of vertices in advance. This allows for fast solving of large-sized inputs, but may not obtain an optimal solution. Furthermore, a solution may not be obtained without additional a vertex. However, it does not explain how to select additional edges that guarantee obtaining not to lose optimal solutions. Hasegawa et al.[4] proposed a heuristic method for practical calculation time. This method can find solutions quickly, but also may not obtain an optimal solution.

In this study, we find and prove four types of *OSP (Optimal Single-Partition)*, that partitions a rectilinear polygon without losing the solution that minimizes the number of feasible rectangles. However, fracturing might not be completed using only *OSP*. Therefore, we propose a method to fracture a polygon in practical calculation time by combining them with greedy partitioning and perform computer experiments. Furthermore, we propose a second method using ILP, since the first method cannot guarantee not to lose optimal solutions due to greedy partitioning. Kahng et al. performed heuristic partitioning prior to ILP, which compromised the optimal solution. Therefore, by using *OSP* before ILP, we can expect to obtain the optimal solution faster than the method using ILP alone. We also perform computer experiments using such a method.

In the next section we describe previous work on the layout fracturing problem. Section 3 presents four types of *OSP* for the problem of fracturing into rectangles and proofs of them. Section 4 compares our fracturing methods that uses four types of *OSP* with conventional methods. Finally, Section 5 gives conclusions and future research directions.

II. PREVIOUS WORKS

Fracturing Problem. Fracture a given rectilinear polygon into the set of rectangles of the minimum number below maximum size limit using only horizontal or vertical line segments.

Hereafter, we call a rectilinear polygon simply *a polygon*. Rectangle below maximum size limit is called *a feasible rectangle*. Maximum size limit is represented by s. An edge on a polygon whose inner angles of its endpoints are both 90° is called *an end-edge*, and a vertex whose inner angle is 270° is called *a concave vertex*.

A. *ILP Based Method[3]*

The objective of this method is to minimize the number of rectangles. Since fracturing a polygon into a set of rectangles eliminates all concave vertices, drawing two line segments from each concave vertex inward of the polygon as candidates of partition line. Create the grid graph whose vertices are the crossings of all line candidates with each other and with the boundary of the polygon, in addition to the original vertices of the polygon, and assign bool variables $x^d(i,j)(d \in \{v,h\},\ i = 1,2,...,hr,$ and $j = 1,...,vr)$ to the edges extending from each vertex v in the horizontal (h) or vertical (v) direction. The variable is set to 1 if the corresponding edge is used for the partition line, and 0 otherwise. Note that variables corresponding to boundary edges are always set to 1 since they belong to any partitioning. The objective is expressed as

$$\text{minimize } \{1 + \sum_{d,i,j} x^d(i,j) - \sum_{i,j} y(i,j)\} \tag{1}$$

To find the number of vertices, a variable $y(i,j)$ is introduced for each vertex which is set to 0 if $v(i,j)$ is isolated, and to 1 otherwise.

$$y(i,j) \le x^h(i-1,j) + x^v(i,j-1) + x^h(i,j) + x^v(i,j) \tag{2}$$

ILP takes super-polynomial time and cannot be obtained in practical execution time for problems larger than a certain size. It has been proposed to pre-segment the input polygons so that the number of vertices is less than a certain number, and then use ILP to perform the segmentation. This speeds up the calculation time, but does not guarantee to obtain the minimum number of rectangles. If a edge in the grid graph is longer than maximum size limit, the graph is modified by adding a new edge as needed. But even though the edges to be added must be chosen appropriately to guarantee an optimal solution obtained, how to select them was not described. In addition, there are cases where the optimal solution cannot be obtained without additional vertices in the first place, but they are not taken into account at all.

B. *Heuristic Method[4]*

Hasegawa et al. proposed a heuristic method, which fractures a polygon into as few feasible rectangles as possible in a practical calculation time considering maximum size limit. For the input polygon P, fracturing is performed in the following two steps.

Step 1. *The horizontal and vertical line segments from the concave vertices of the polygon P are used to create irregularly spaced grids within P. By selecting partition lines from among these line segment candidates, P is fractured into a set of rectangles.*

Step 2. *Fracture rectangles larger than maximum size limit into the set of feasible rectangles.*

III. *OPTIMAL SINGLE-PARTITION(OSP)*

For the problem of fracturing a rectilinear polygon, Hotta et al. defined *OSP*[5], which partitions a rectilinear polygon without losing the solution with the minimum number of rectangles, as follows.

Optimal Single-Partition: Let $X(R_0)$ be the number of feasible rectangles when the polygon R_0 is fractured into the minimum number of feasible rectangles. When polygon R_0 is partitioned into polygons R_1 and R_2, if $X(R_0) = X(R_1) + X(R_2)$, then this partition is called *Optimal Single-Partition*.

Firstly, we present a lemma to make the proof concise.

Lemma 1. *Let R_1 be the remaining polygon of R after cutting off one side by a line segment L, which is horizontal or vertical. If R is fractured into the minimum number of feasible rectangles, then the number of feasible rectangles that are included even partially in R_1 is always not less than $X(R_1)$.*

Proof. Let OPT be one of the feasible rectangle sets where the input polygon R is fractured into the minimum number. Let N be the set of feasible rectangles in OPT that are at least partially contained in R_1. If outside of R_1 is cut off by L for all feasible rectangles in N, then the polygon formed by the union of all remaining feasible rectangles coincides with R_1. Since cutting off a rectangle by a horizontal or vertical line segment always results in a rectangle, the number of feasible rectangles remains the same as $|N|$. Therefore $|N| \ge X(R_1)$. □

The followings are the necessary and sufficient conditions for four types of *OSP* for rectangular fracturing of polygons.

A. *Edge Focus Type OSP*

Theorem 1. *If there is a location on one of the sides that shares an endpoint with end-edge E_{tip} that is s away from E_{tip}, the polygon is divided by a line segment that starts at this location, extends inside the polygon parallel to E_{tip} and ends at the crossing with the polygon's perimeter. If the polygon containing the edge E_{tip} is rectangular or L-shaped,*

$$\lceil \frac{(E_{tip}length)}{s} \rceil = \lceil \frac{(cutlinelength)}{s} \rceil + \lceil \frac{(E_{tip}length) - (cutlinelength)}{s} \rceil$$

and the length of the dividing line is not longer than the length of the edge E_{tip}, then this partitioning is OSP (see Fig. 1).

Proof. Perform *Edge focus Type OSP* of a polygon R_0 by focusing on a certain end-edge E_{tip}. Denote the cutline partitioning R_0 into R_1 and R_2 as d_1. Assume that polygon R_1 containing E_{tip} is a rectangle or L-shape but d_1 is not *OSP*. The relation between the number of feasible rectangles of R_0, R_1 and R_2 is as follows.

$$X(R_0) < X(R_1) + X(R_2) \tag{3}$$

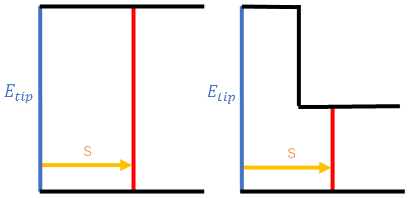

Fig. 1. Example of *Edge Focus Type OSP*

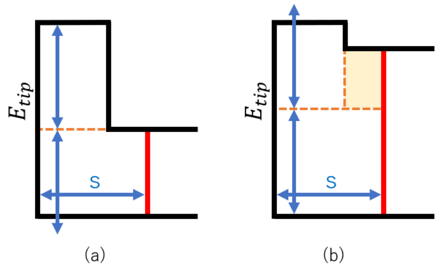

Fig. 2. (a)Example of $\lceil\frac{(E_{tip}length)}{s}\rceil = \lceil\frac{(cutlinelength)}{s}\rceil + \lceil\frac{(E_{tip}length)-(cutlinelength)}{s}\rceil$. (b)Example of $\lceil\frac{(E_{tip}length)}{s}\rceil <$ $\lceil\frac{(cutlinelength)}{s}\rceil + \lceil\frac{(E_{tip}length)-(cutlinelength)}{s}\rceil$(Contains rectangles not tangent to E_{tip}).

Let OPT be any one of the feasible rectangle sets where the input polygon R_0 is fractured into the minimum number. Considering OPT, there is always at least one feasible rectangle spanning d_1 in OPT. Let N_1 be the set of feasible rectangles in OPT that are completely contained in R_1, and N_2 be the others. This gives

$$|N_1| + |N_2| = X(R_0) < X(R_1) + X(R_2) \tag{4}$$

Considering the polygon that combines all the rectangles of N_1, this contains all the areas tangent to E_{tip}. This is because the feasible rectangle across d_1 cannot be tangent to E_{tip} by $\lceil\frac{(E_{tip}length)}{s}\rceil = \lceil\frac{(cutlinelength)}{s}\rceil + \lceil\frac{(E_{tip}length)-(cutlinelength)}{s}\rceil$(see Fig. 2). Therefore, its width is less than s and its height is E_{tip}, and the relation of the number of feasible rectangles is as follows.

$$|N_1| = \lceil\frac{(E_{tip}length)}{s}\rceil = X(R_1) \tag{5}$$

For all the feasible rectangles in N_2, cut off the side of R_1 by cutline d_1. From **Lemma 1**,

$$X(R_2) \leq |N_2| \tag{6}$$

From equation (5) and equation (6), we obtain that

$$|N_1| + |N_2| \geq X(R_1) + X(R_2) \tag{7}$$

which contradicts the equation (4). Therefore, it is proved that cutline d_1 is *OSP*. □

B. *L-Shape Cutout Type OSP*

Theorem 2. *A polygon is divided into two parts by a straight line passing through the concave vertex, and the partition is OSP if both the width and height of one polygon are less than s, and no perpendicular line from the concave vertex of the L-shape to the partition line (including both ends) can be drawn (see Fig. 3).*

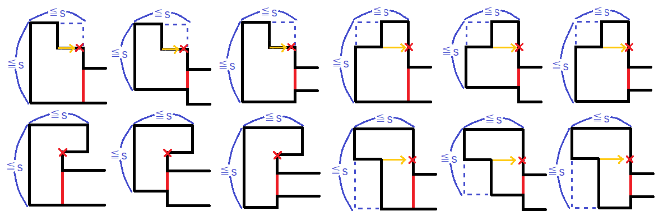

Fig. 3. Example of *L-Shape Cutout Type OSP*. The red line is the dividing line for *L-Shape Cutout Type OSP*.

Proof. Partition the polygon R_0 into R_1 and R_2 by cutline d_1 satisfying the condition of *L-Shape Cutout Type OSP*. Assume that R_1 is L-shape whose width and height are both less than s but d_1 is not *OSP*. The relation between the number of feasible rectangles in R_0, R_1 and R_2 is as follows.

$$X(R_0) < X(R_1) + X(R_2) \tag{8}$$

Let OPT be any one of the feasible rectangle sets where the input polygon R_0 is fractured into the minimum number. Considering OPT, there is always at least one feasible rectangle spanning d_1 in OPT. Let N_1 be the set of feasible rectangles in OPT that are completely contained in R_1, and N_2 be the others. This gives

$$|N_1| + |N_2| = X(R_0) < X(R_1) + X(R_2) \tag{9}$$

Since R_1 is L-shape whose width and height are both less than s, $X(R_1) = 2$. Here, Fig. 3 shows that $|N_1| \geq 2$, thus the relation between $|N_1|$ and $X(R_1)$ is

$$|N_1| \geq X(R_1) \tag{10}$$

For all the feasible rectangles in N_2, cut off the R_1 side by cutline d_1. From **Lemma 1**,

$$|N_2| \geq X(R_2) \tag{11}$$

Then, from equations (10) and (11),

$$|N_1| + |N_2| \geq X(R_1) + X(R_2) \tag{12}$$

can be derived. The equation (12) contradicts the equation (9). Therefore, it is proved that cutlined_1 is *OSP*. □

C. *Rectangle Cutout Type OSP*

Theorem 3. *When the other endpoint of one of the sides that shares an endpoint with the edge E_{tip} is a concave vertex, the polygon is divided by a line segment that starts*

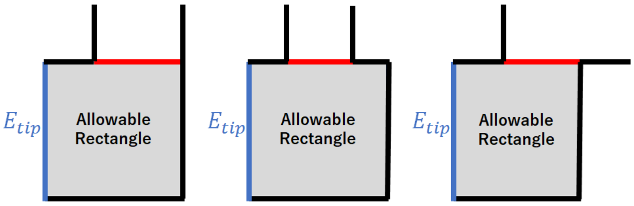

Fig. 4. Example of *rectangle cutout type OSP*

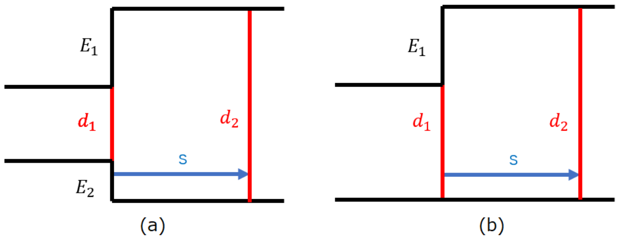

Fig. 5. Example of *Alley Cut-Off Type OSP*. The red line d_1 is the dividing line. (a) When $E_2 \neq 0$ (b) When $E_2 = 0$.

at this point, extends perpendicular to E_{tip} inside the polygon, and ends at its crossing with the polygon's perimeter. If the polygon containing edge E_{tip} is an admissible rectangle, then This partition is OSP. (see Fig.4).

Proof. Perform *Rectangle Cutout Type OSP* of a polygon R_0 by focusing on a certain end-edge E_{tip}. Denote the cutline partitioning R_0 into R_1 and R_2 as d_1. Assume that polygon R_1 is a feasible rectangle containing E_{tip} but d_1 is not *OSP*. The relation between the number of feasible rectangles of R_0, R_1 and R_2 is as follows.

$$X(R_0) < X(R_1) + X(R_2) \tag{13}$$

Let OPT be any one of the feasible rectangle sets where the input polygon R_0 is fractured into the minimum number. Considering OPT, there is always at least one feasible rectangle spanning d_1 in OPT. Let N_1 be the set of feasible rectangles in OPT that are completely contained in R_1, and N_2 be the others. This gives

$$|N_1| + |N_2| = X(R_0) < X(R_1) + X(R_2) \tag{14}$$

Considering the polygon that combines all the rectangles of N_1, this contains all the areas tangent to E_{tip}, then $|N_1| \geq 1$. This is because d_1 is perpendicular to E_{tip} and the feasible rectangle across d_1 cannot be tangent to E_{tip}(see Fig. reffig:8). Here, since R_1 is a feasible rectangle, $X(R_1) = 1$. Therefore, the relation of the number of feasible rectangles is as follows.

$$|N_1| \geq X(R_1) \tag{15}$$

For every feasible rectangle in N_2, cut off the side of R_1 by cutline d_1. From **Lemma 1**,

$$|N_2| \geq X(R_2) \tag{16}$$

From equation (15) and equation (16), we obtain that

$$|N_1| + |N_2| \geq X(R_1) + X(R_2) \tag{17}$$

which contradicts the equation (14). Therefore, it is proved that cutline d_1 is *OSP*. □

D. Alley Cut-Off Type OSP

Theorem 4. *Divide the polygon into two parts by a straight line segment d_1 passing through the concave vertex. For an end-edge E_{tip} including d_1 completely, if there is a point s away from E_{tip}, a line segment starting from this point is extended inside the polygon parallel to E_{tip}. The endpoint is where it intersects the outer circumference of the polygon, and divide the polygon by this line segment d_2. For the part of E_{tip} excluding d_1, let E_1 be the part adjacent to the starting point of d_1 and E_2 be the part adjacent to the ending point. If the polygon containing the end-edge E_{tip} is a rectangle and satisfies*

$$\left\lceil\frac{(E_{tip}length)}{s}\right\rceil = \left\lceil\frac{(E_1 length)}{s}\right\rceil + \left\lceil\frac{(E_2 length)}{s}\right\rceil$$

then this partition is OSP (see Fig. 5).

Proof. Partition the polygon R_0 into R_1 and R_2 by cutline d_1 satisfying the condition of *Alley Cut-Off Type OSP*, and R_2 into R_2a and R_2b by cutline d_2. Assume that R_2a is a rectangle containing both d_1 and d_2, but d_1 is not *OSP*. The relation between the number of feasible rectangles of R_0, R_1 and R_2 is as follows.

$$X(R_0) < X(R_1) + X(R_2) \tag{18}$$

Let OPT be any one of the feasible rectangle sets where the input polygon R_0 is fractured into the minimum number. Considering OPT, there is always at least one feasible rectangle spanning d_1 in OPT. Let N_1 be the set of feasible rectangles in OPT that are partially contained in R_1, N_2 be the set of feasible rectangles that are completely contained in R_{2a}, and N_3 be the set of feasible rectangles that are partially contained in R_{2b}. Since d_1 and d_2 are exactly s apart, N_1 and N_3 never contain the same feasible rectangle. This gives the relation between the number of feasible rectangles as

$$|N_1| + |N_2| + |N_3| = X(R_0) < X(R_1) + X(R_2) \tag{19}$$

For every feasible rectangle in N_1, cut off the side of R_2 by cutline d_1. From **Lemma 1**,

$$|N_1| \geq X(R_1) \tag{20}$$

Considering a polygon that combines all the feasible rectangles of N_2, the feasible rectangle spanning d_1 and the feasible rectangle spanning d_2 cannot be tangent to either E_1 or E_2 because of the span s (see Fig. 5). Therefore, this polygon contains all E_1 and E_2 tangent parts, with width less than s and height $(E_1 length)$ and $(E_2 length)$, respectively.

$$|N_2| \geq \lceil (E_1 length)/s \rceil + \lceil (E_2 length)/s \rceil \quad (21)$$

Under the condition $\lceil (E_{tip} length)/s \rceil = \lceil (E_1 length)/s \rceil + \lceil (E_2 length)/s \rceil$, the relation of the number of feasible rectangles is

$$|N_2| \geq \lceil (E_{tip} length)/s \rceil \quad (22)$$

Since $X(R_{2a}) = \lceil (E_{tip} length)/s \rceil$, then

$$|N_2| \geq X(R_{2a}) \quad (23)$$

For every feasible rectangle in N_3, cut off the side of R_2a by cutline d_2. From **Lemma 1**,

$$|N_3| \geq X(R_{2b}) \quad (24)$$

From equations (20), (23) and (24),

$$|N_1| + |N_2| + |N_3| \geq X(R_1) + X(R_{2a}) + X(R_{2b}) \quad (25)$$

For R_2, cutline d_2 is satisfying the condition of *Edge focus Type OSP*, so

$$X(R_2) = X(R_{2a}) + X(R_{2b}) \quad (26)$$

From equations (25) and (26) the following is obtained, but it contradicts equation (19).

$$|N_1| + |N_2| + |N_3| \geq X(R_1) + X(R_2) \quad (27)$$

Therefore, it is proved that cutline d_1 is *OSP*. □

IV. EXPERIMENTAL RESULTS

We propose *OSP Fracturing with greedy* that partitions a polygon as much as possible by *OSP* of *Edge focus Type*, *L-Shape Cutout Type*, *Rectangle Cutout Type*, and *Alley Cut-Off Type*. When there is no more *OSP*, selects a line segment that does not increase the number of feasible rectangles among the straight segments starting from concave vertices, then uses *OSP* again. Greedy partition is chosen from candidates of partition line starting from concave vertices for reducing the number of concave vertices of a polygon. As an index for determining which line segment to choose, we used the same cost value as in Hotta's method[5], and greedy partition was performed using the slice line with the lowest cost value. We use the IBM ILOG CPLEX Optimization Studio 22.1.0[6] to solve all ILP instances. All tests run on a computer with an Intel(R) Core(TM) i5-1240P CPU and 16 GB memory.

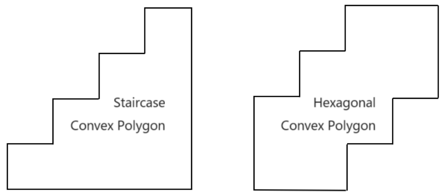

Fig. 6. Examples of regular convex polygons used for input. Staircase convex polygon (#concave vertices is 3) and hexagonal concave polygon (#concave vertices is 4).

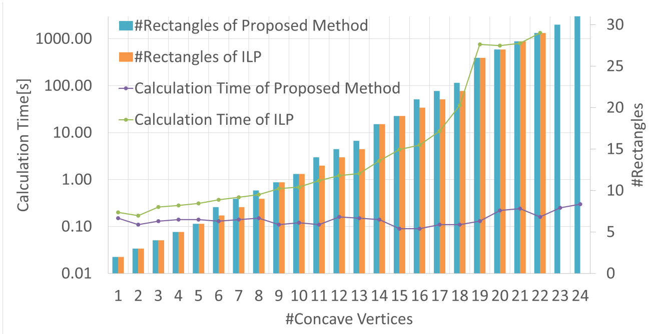

Fig. 7. Comparison of *OSP Fracturing with greedy* to ILP for the number of concave vertices of a staircase convex polygon.

A. Comparison with ILP method

This experiment was done to see how much faster our method is than ILP and how close its solution is to the optimal solution. Input data are two types of convex polygons(see Fig. 6) with regularly increasing number of vertices. Maximum size limit s is set at 50. Results are shown in Fig.7 and Fig. 8.

It can be seen that ILP takes an explosive amount of time as the number of concave vertices increases. In other words, fast solving by ILP is possible only when the number of concave vertices is quite small. On the other hand, our method provides near-optimal solutions at high speed,

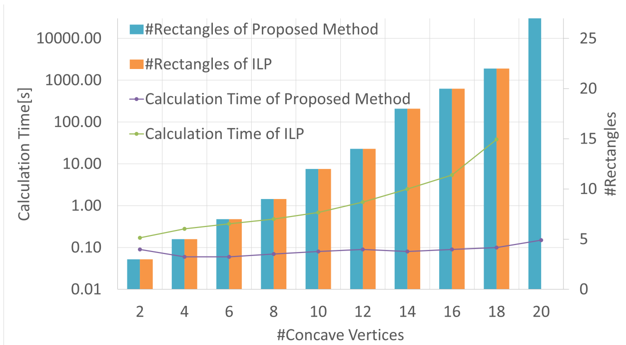

Fig. 8. Comparison of *OSP Fracturing with greedy* to ILP for the number of concave vertices of a hexagonal convex polygon.

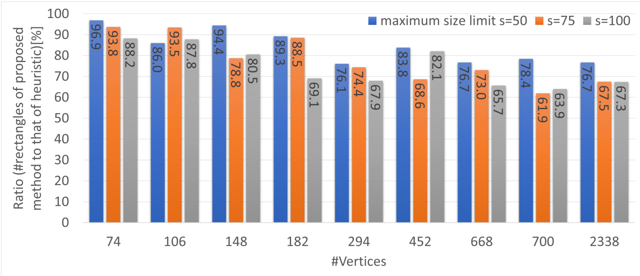

Fig. 9. The percentage of #rectangles obtained by the proposed method for 9 different polygons, when #rectangles obtained by the heuristic method is 100%. Experiments were conducted by varying maximum size limit s to 50, 75, and 100.

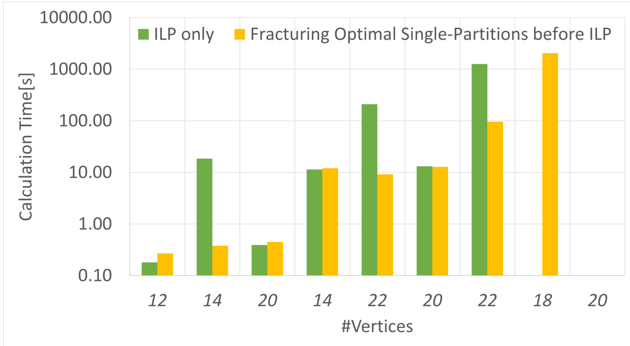

Fig. 10. Comparison of *OSP Fracturing before ILP* and ILP in fracturing 9 polygons with a small number of vertices.

regardless of the number of concave vertices. Furthermore, ILP can not obtain the solution due to out of memory at 23 concave vertices for the staircase type and at 20 concave vertices for the hexagonal type. Our method can find solutions for even more concave vertices.

B. *Comparison with heuristic method*

Next, we conducted an experiment to see how much our method could reduce the number of rectangles compared to heuristic method in a practical calculation time. Our method is compared to heuristic method[4] and to the case of greedy partitioning only. For the results of this experiment, Fig. 9 shows what percentage of solutions our method obtained over the heuristic method. The inputs were 9 different polygons and $s = 50, 75, 100$.

- greedy partitioning
- heuristic method[4]
- *OSP Fracturing with greedy*

It can be seen that our method obtains the best solution in most cases. Note that since the computation times were all less than one minute, no comparison was made.

C. *OSP Fracturing before ILP*

In *OSP Fracturing with greedy*, it does not guarantee obtaining an optimal solution due to greedy partitioning. Therefore, we propose *OSP Fracturing before ILP*, using ILP to fracture remaining polygons when there is no more *OSP* for the input polygon. It is expected that the optimal solution may be obtained faster than when only ILP is used. Fig. 10 shows an experiment comparing *OSP Fracturing before ILP* with ILP only. Since the computation requires super-polynomial time, 9 different polygons with a small number of vertices are used. Maximum size limit is set at $s = 25$. Note that the computation time limit was set to 2 hours.

The experimental results show that *OSP Fracturing before ILP* does not necessarily reduce the computation time compared to ILP. However, in some cases, it is able to obtain the optimal solution in a short time for problems that would take an explosive amount of time with ILP. For the eighth input, the ILP did not find a solution within the time limit, while the proposed method did. For the ninth input, neither method could find a solution.

V. CONCLUSIONS

We have found and proved four types of *OSP* for the fracturing problem. Since these are partitions that do not lose the optimal solution, they can be used in combination with greedy partitioning or ILP to achieve both high speed and good quality. Computer experiments confirmed that using *OSP*, in many cases, is faster than ILP and obtains a smaller number of rectangles than heuristic method. Furthermore, we confirmed that in some cases, *OSP Fracturing before ILP* can save a significant amount of time compared to simply solving with ILP. The actual input could be a polygon with holes, about which we expect partitions like "Alley Cut-Off Type OSP" that cuts polygons to be of great help.

REFERENCES

[1] S. Matsui, Y. Ochiai, and H. Yamashita, "Present state and prospect of electron beam lithography," *OYOBUTURI*, vol. 70, 4, pp. 411–417, 2001.

[2] C. Spence, S. Goad, P. Buck, R. Gladhill, R. Cinque, J. Preuninger, Ü. Griesinger and M. Blöcker, "Mask data volume: historical perspective and future requirements," *22nd European Mask and Lithography Conference*, vol. 6281, pp. 62810H, 2006.

[3] A. B. Kahng, X. Xu, and A. Zelikovsky, "Yield- and cost-driven fracturing for variable shaped-beam mask writing," *24th Annual BACUS Symposium on Photomask Technology*, vol. 5567, pp. 360–371, 2004.

[4] M. Hasegawa, and K. Fujiyoshi, "Polygon fracture method considering maximum shot size for variable shaped-beam mask writing," *IEICE Conf. '16*, vol. 115, no. 239, CAS2015-34, pp. 69–74, 2016.

[5] T. Hotta, and K. Fujiyoshi, "An Improved Method of Fracturing into Rectangles and L-Shapes Obeying Size Limits," *Proc. IEICE Gen. Conf. '21*, p. A–1–1, 2021.

[6] IBM ILOG CPLEX Optimization Studio 22.1.0, http://www.cplex.com/