

# Evaluating Signal Integrity in InFO Package via Learning-based Methods

Yi-Hua Yeh

International College of Semiconductor Technology  
National Yang Ming Chiao Tung University  
Hsinchu, Taiwan  
Email: yhyeh.st11@nycu.edu.tw

Hao-Ju Chang, Hung-Ming Chen,  
Chien-Nan Liu

Institute of Electronics and SoC Center  
National Yang Ming Chiao Tung University  
Hsinchu, Taiwan  
Email: {s410510026.ee10, hmchen, jimmyliu}@nycu.edu.tw

**Abstract—** This work investigates further signal integrity issues in the InFO package, focusing on high-bandwidth memory. Ensuring signal integrity is crucial in layout design to accommodate the increasingly complex modern circuit designs, particularly in high-speed signal transmission. This research proposes regression prediction models based on CatBoost, XGBoost, and LightGBM to predict the values of signal quality indicators such as eye height and eye width, providing a methodology to assess signal integrity. In this work, we use the systematic method of multi-layer modeling and angle simulation, signal integrity in design has been quantitatively characterized and optimized. Such a framework demonstrates high accuracy in evaluating and optimizing signal integrity of InFO packages through rapid evaluation.

## I. INTRODUCTION

High-speed and high-density interconnect makes it necessary to find new ways to ensure Signal integrity (SI) [1], especially with the emergence of advanced electronic systems requiring high-speed system solutions. While SI has become more important with advances in integrated circuit (IC) technology, traditional methods for maintaining it are falling short. As a result, learning-based methodologies have become popular in solving the emerging problems of SI.

Several researchers have explored machine learning to model and optimize high-speed channels. For example, [2] demonstrated that deep neural networks (DNNs) outperform support vector regression (SVR) in capturing the non-linear characteristics of high-speed channels. Building on this, [3] combined DNNs with genetic algorithms, significantly enhancing the efficiency of SI optimization. Further advancing SI analysis, [4] introduced a semi-supervised hybrid neural network (HNN) capable of accurately forecasting eye-diagram metrics with minimal labeled data. Complementarily, [5] developed an anomaly detection system using machine learning, which enabled automatic SI analysis even without detailed circuit information.

Addressing the computational complexities in high-speed channel design, [6] created an active subspace-based SVR for rapid sensitivity analysis. In parallel, [7] investigated AI/ML-infused PCB design systems, highlighting their utility

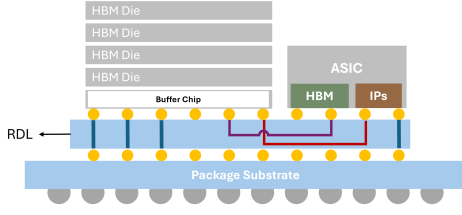
in enabling early-stage SI-compliant decisions. In the realm of transmission line design, [8] introduced a lifelong learning architecture for inverse design, offering a transformative approach to SI-aware design. Meanwhile, [9] explored the combined challenges of SI and power integrity (PI) in advanced packaging technologies. To further enhance SI, [10] proposed a machine learning-based (DNN and CatBoost [11]) integrated fan-out (InFO) routing flow that successfully improved SI on HBM3 benchmarks.

While previous works have mainly focused on 1D routing and high-speed channel modeling, this study targets the growing relevance of the InFO package in modern electronic designs. This research proposes a new angle-bending multilayer considered model for eye diagram indicators prediction. This new method provided an improved evaluation of routing layout quality, which is necessary whether the goal is to optimize signal integrity in large or complex routing scenarios that may be beyond the traditional SI modeling.

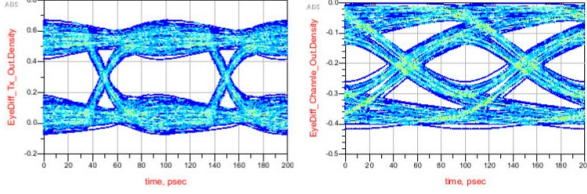
The proposed model is based on a multiple-layer staggered shielding technique reinforced with an angle bending effect. It is designed to be incorporated into SI simulation to improve the accuracy of predicting eye width and eye height. This enhancement enables better design of high-speed interconnects in the InFO package, ensuring good SI performance over complex layouts that demand an optimized interconnecting routing strategy.

## II. SIGNAL INTEGRITY ANALYSIS FOR HBM3 AND INFO PACKAGE

Signal integrity (SI) is critical in modern high-performance computing systems, especially in applications involving high-bandwidth memory (such as HBM3) and integrated fan-out (InFO) packaging (Fig. 1). The InFO technology offers significant advantages by reducing RC delay and eliminating the need for TSVs, thereby improving signal transmission at high data rates. Recent advancements in machine learning methods, particularly gradient boosting techniques like XGBoost and LightGBM, have further enhanced the ability to predict and optimize signal integrity metrics such as eye diagrams. This section reviews related work in InFO packaging, SI optimization, and machine learning frameworks.



**Fig. 1: Advanced packaging: InFO and HBM3.**



**Fig. 2: Eye diagrams (a) opened-eye (b) closed-eye.**

#### A. Signal Integrity in InFO Packaging

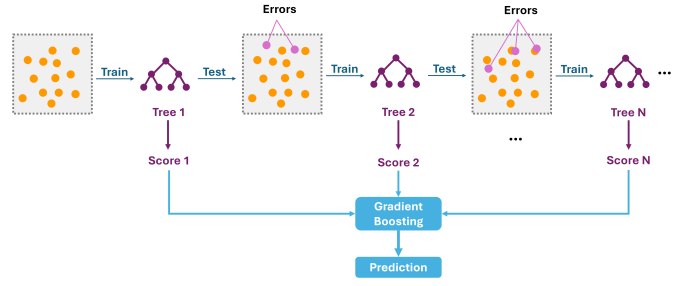
InFO technology, a form of Fan-Out Wafer-Level Packaging (FOWLP), has emerged as a critical enabler for high-performance computing and mobile applications due to its high interconnect density and bandwidth [12]. Using a fine redistribution layer (RDL), InFO eliminates the need for TSVs, commonly required in 3D integration, and significantly reduces RC delay. These properties are crucial for signal integrity in systems such as HBM3, where maintaining low latency and high bandwidth is essential.

A recent study [12] has shown that InFO packaging provides superior signal integrity, particularly in mitigating crosstalk, a significant source of signal degradation at higher data rates. The enhanced layout flexibility the fine RDL provides allows for optimized routing, minimizing impedance mismatch, and improving the eye diagram performance, a critical metric for data transmission evaluation in SI. Fig.2 illustrates the impact of optimized layouts on eye diagram performance, demonstrating the difference between open and closed eye diagrams. An opened-eye diagram indicates minimal interference and better signal quality, whereas a closed-eye diagram suggests significant distortion or noise.

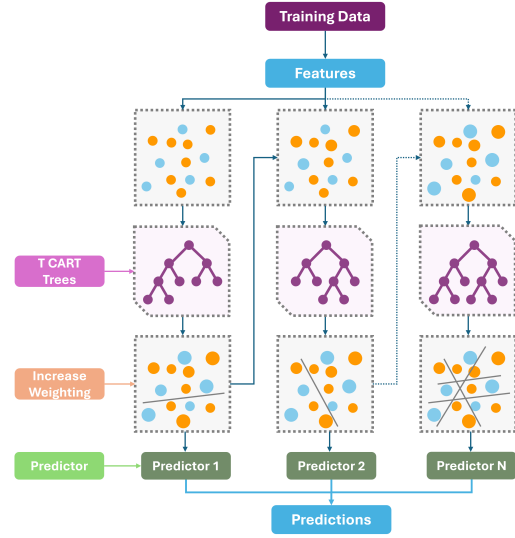
#### B. Machine Learning for Signal Integrity Prediction

In recent years, machine learning methods such as gradient boosting have become powerful tools in predicting and optimizing values, which is suitable for predicting SI in high-speed interconnects. These methods, including XGBoost and LightGBM, are particularly effective for handling complex regression tasks involving SI indicators like eye height (EH) and eye width (EW).

XGBoost [13] builds upon the traditional gradient boosting tree (Fig. 3) by incorporating regularization terms to reduce overfitting. This is particularly useful in SI problems, where overfitting can lead to poor generalization across signal routing configurations. Fig. 4 illustrates XGBoost.

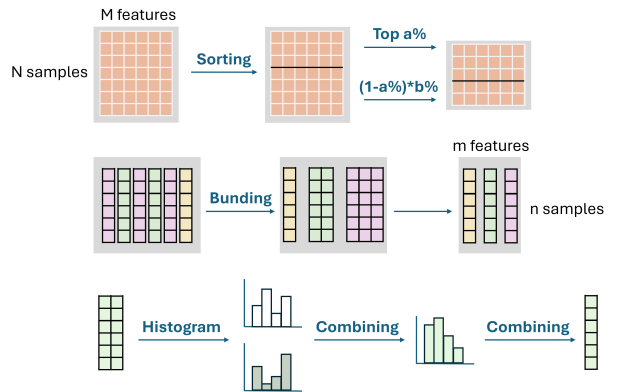


**Fig. 3: Gradient boosting.**



**Fig. 4: XGBoost.**

Similarly, LightGBM [14] (Fig. 5), with its histogram-based split search and leaf-wise growth strategy, offers faster training times and reduced memory consumption, making it suitable for large SI datasets.



**Fig. 5: LightGBM.**

Studies have demonstrated the effectiveness of these algorithms in predictions, outperforming traditional regression techniques in terms of both speed and accuracy. These methods



**Fig. 6: High speed circuit design: transmitter and receiver.**

also enable the rapid evaluation of various layout configurations, providing designers with valuable insights into how signal routing affects SI.

### C. Optimization Techniques for Signal Integrity

In addition to machine learning approaches, optimization techniques are critical in improving signal integrity. The Gurobi Optimizer [15] is widely utilized for solving complex optimization problems, and it excels in handling mixed integer programming and multi-objective optimization, making it suitable for applications that require minimizing SI degradation while adhering to strict design constraints.

For instance, optimizing the layout of signal and ground wires in InFO packaging involves balancing multiple objectives, such as minimizing crosstalk and ensuring minimal impedance mismatch. Gurobi's ability to handle non-convex objective functions and large datasets makes it a valuable tool in SI optimization, allowing for fine-tuning layout parameters to achieve optimal signal integrity.

## III. MODEL CONSTRUCTION

This research addresses the critical issue of signal integrity (SI) in modern high-speed circuit designs (Fig. 6). Routing layout parameters—such as spacing, angle, and wire length—significantly influence key SI performance indicators like eye height (EH) and eye width (EW), which are crucial for evaluating signal quality. As circuit complexity increases, accurately predicting these metrics becomes essential to avoid signal degradation due to crosstalk, impedance mismatches, or reflection. This work aims to develop predictive models capable of quantifying the impact of these layout parameters on SI, enabling efficient optimization during the design process.

### A. Signal Integrity Simulations

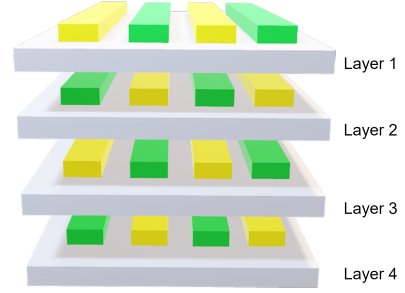
We conducted SI simulations using Keysight Advanced Design System (ADS) software to evaluate the effects of different layout configurations. The simulations focused on single-layer and multi-layer structures and varying wire angles to assess how electromagnetic coupling and geometric factors affect signal quality.

#### • Multi-Layer Simulation:

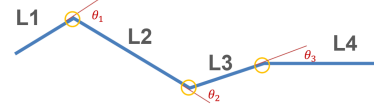
We simulated signal transmission in multi-layer structures (Fig. 7) to understand the impact of inter-layer electromagnetic coupling. We examined the signal interference between layers and the shielding effects in double-layer and triple-layer configurations.

#### • Angle Simulation:

We investigated how varying bend angles in transmission lines with fixed and variable lengths influence EH and



**Fig. 7: Multiple layers in signal transmission.**



**Fig. 8: Bend positions and angles.**

EW (Fig.8). The objective was to understand how these geometric parameters contribute to SI degradation and guide optimal design.

#### – Same Length Simulation:

We consider the fixed lengths of the transmission lines but that the transmission line's angle changes with different positions, i.e.,  $(\theta_1, \theta_2, \theta_3)$ . We attempted to observe how these angle changes affect EH and EW specifically.

#### – Different Length Simulation:

The simulation explores the effects of varying angles on transmission lines with different lengths. By cross-referencing these two variables, we can assess how length and angle harmonize to influence SI.

This design leads to a better understanding of the optimization for SI performance regarding transmission line design, which will help specify the best layout for maintaining high-quality signals.

### B. Mixed-Integer Linear Programming (MILP) Model

We developed a mixed integer linear programming (MILP) model to model the effect of layout parameters on SI mathematically. The formula incorporates factors such as spacing ( $D$ ), placement ( $P$ ), wire length ( $L$ ), and relative angle ( $\theta$ ) to predict EH. The equation is as follows:

$$EH = \alpha_0 + \sum_{i,j} (\alpha_{ij} \cdot D_{ij} + \beta_{ij} \cdot P_{ij} + \gamma_{ij} \cdot L_{ij} + \delta \cdot \theta) + \epsilon \quad (1)$$

Where:

- $D_{ij}$  represents the distance between signal lines and surrounding lines,
- $P_{ij}$  denotes the relative positions of the signal lines,
- $L_{ij}$  is the wire length,

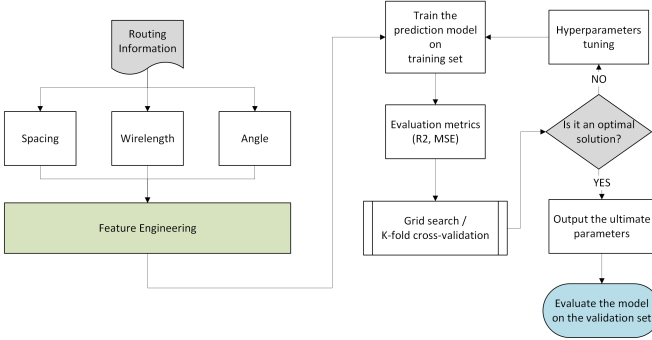


Fig. 9: Training flow.

- $\theta$  is the bend angle,
- $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta$  are the optimized coefficients, and
- $\epsilon$  is the error term.

In this formulation, variables related to routing layer assignment are modeled as integers, while parameters such as spacing ( $D_{ij}$ ), bend angle ( $\theta$ ), and wire length ( $L_{ij}$ ) are continuous variables.

The MILP model is applied to determine the optimal combination of routing layer selection, spacing, and angle values that maximize the eye height (EH) under given physical and design constraints. The integer variables correspond to discrete routing and layer assignments, while continuous variables capture geometric and electrical parameters.

The coefficients are optimized using Gurobi to minimize the sum of squared errors (SSE), improving the accuracy of the electromagnetic interference prediction.

The optimization problem can be described as follows:

$$\text{minimize } SSE = \sum (EH_{\text{observed}} - EH_{\text{predicted}})^2 \quad (2)$$

### C. Predictive Model Training

The predictive models were trained using data from different layout configurations (Fig. 9), including Ground-Signal-Ground (GSG), Signal-Ground-Signal-Ground (SGSG) (Figs. 10, 11, 12), and multi-layer structures. The dataset was preprocessed using a robust scaler to standardize features, followed by feature extraction based on layout properties such as spacing, angle, and wire length.

Eq.(1) represents the optimization-based framework, while the machine learning algorithms (CatBoost, XGBoost, LightGBM) form a separate predictive framework trained on simulation data to directly predict EH and EW.

We employed machine learning algorithms, including CatBoost, XGBoost, and LightGBM, to develop predictive models for EH and EW. These models underwent cross-validation to ensure robustness, and their hyperparameters were fine-tuned for optimal performance. By integrating SI simulations, MILP-based formula modeling, and machine learning techniques, this methodology provides a comprehensive approach to evaluating and predicting signal integrity during circuit design.

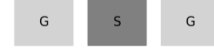


Fig. 10: Layout configuration: GSG.



Fig. 11: Layout configuration: SGSG.



Fig. 12: Layout configuration: SGSGS.

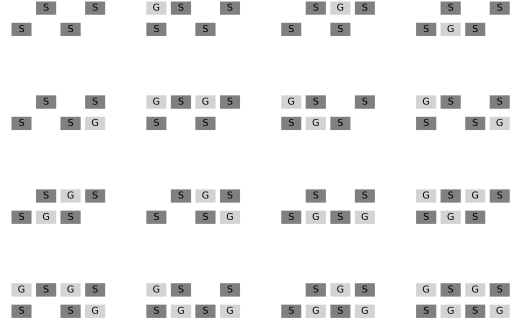


Fig. 13: Double layer configurations.



Fig. 14: Triple layer configurations.

## IV. EXPERIMENTAL RESULTS AND DISCUSSION

This section presents the results of signal integrity (SI) simulations conducted using ADS for various routing configurations and layer structures (Fig. 15). We also evaluate machine learning (ML) models for predicting eye height and width, essential metrics for SI assessment, across different transmission line layouts. Finally, we highlight the significance of our method for efficient SI evaluation in IC design.

TABLE I: Summary of Datasets

|            | GSG  | SGSG | SGSGS | 2_layer   | 3_layer  |
|------------|------|------|-------|-----------|----------|
| Spacing_1  | O    | O    | O     | O         | O        |
| Spacing_2  | O    | O    | O     | O         | O        |
| Spacing_3  | X    | O    | O     | O         | O        |
| Spacing_4  | X    | X    | O     | X         | X        |
| Category   | X    | X    | X     | 16        | 4        |
| Datapoints | 1404 | 4212 | 12636 | 4212 * 16 | 4212 * 4 |

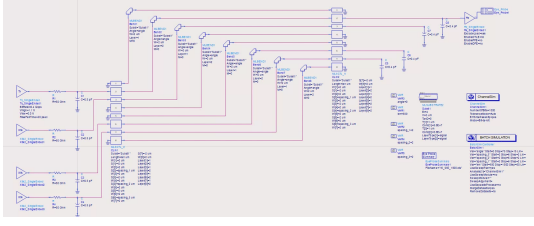


Fig. 15: Simulation eye diagram using ADS.

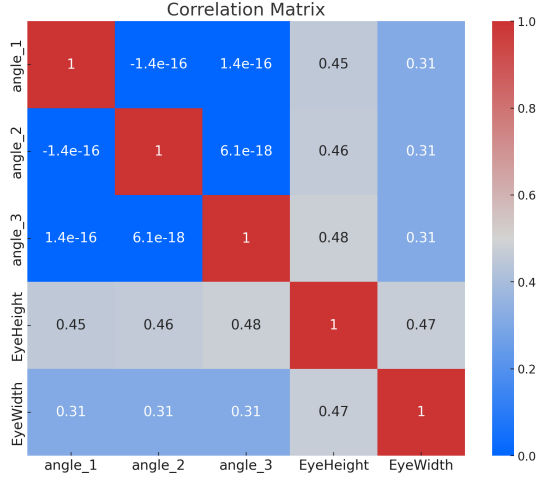


Fig. 16: Relationship of angles in different positions.

The dataset in Table I was generated by systematically varying trace angles from  $0^\circ$  to  $75^\circ$  in  $15^\circ$  increments, trace wire lengths from  $500 \mu\text{m}$  to  $3000 \mu\text{m}$  in  $100 \mu\text{m}$  increments, and spacing between  $2 \mu\text{m}$  and  $6 \mu\text{m}$  in  $2 \mu\text{m}$  increments. Each configuration was simulated in Keysight ADS using the same HBM3 channel model.

#### A. Simulation Results

We performed SI simulations for routing configurations—GSG, SGSG, and SGSGS—under single, double, and triple-layer structures. The eye diagrams (Fig. 15) demonstrate the variation in signal performance due to routing geometry. Eye height (EH) and eye width (EW) were computed for each scenario to quantify SI.

Key observations from the simulation include:

- **Single Layer:**  
As seen in Fig. 10, Fig. 11, and Fig. 12, variations in signal path spacing significantly impact both EH and EW. GSG shows higher EH than SGSGS due to the deteriorating shielding effects of the additional signal lines.
- **Double Layer:**  
Fig. 13 shows the shielding effect, where fully shielded configurations have the highest EH due to minimal signal crosstalk.
- **Triple Layer:**  
As shown in Fig. 14, asymmetric configurations lead to

reduced EH, with the best performance in symmetric, fully-shielded setups.

The correlation matrix in Fig. 16 reveals a moderate to strong positive relationship between signal angles and EH, confirming that geometric factors like angle and spacing influence SI metrics.

#### B. Predictive Modeling Results

TABLE II: Evaluating Gurobi Formula Accuracy

| Actual vs. Predicted | GSG    | SGSG   | SGSGS  | Double layer | Triple layer |
|----------------------|--------|--------|--------|--------------|--------------|
| R <sup>2</sup>       | 0.8739 | 0.8122 | 0.6159 | 0.7105       | 0.9084       |
| Correlation          | 0.9348 | 0.9012 | 0.7848 | 0.8429       | 0.9531       |

TABLE III: Accuracy of Different Prediction Models

|         |    | CatBoost[11]   |        | LightGBM       |        | XGBoost        |        | Gurobi         |        |
|---------|----|----------------|--------|----------------|--------|----------------|--------|----------------|--------|
|         |    | R <sup>2</sup> | MSE    | R <sup>2</sup> | MSE    | R <sup>2</sup> | MSE    | R <sup>2</sup> | MSE    |
| GSG     | EH | 0.9998         | 0.0006 | 0.9996         | 0.0008 | 0.9992         | 0.0012 | 0.8739         | 0.0002 |
|         | EW | 0.9459         | 0.1481 | 0.8305         | 0.2621 | 0.8222         | 0.2685 | -              | -      |
| SGSG    | EH | 0.9998         | 0.0005 | 0.9995         | 0.0008 | 0.9992         | 0.0010 | 0.8122         | 0.0003 |
|         | EW | 0.9940         | 0.1812 | 0.9844         | 0.2924 | 0.9851         | 0.2857 | -              | -      |
| SGSGS   | EH | 0.9992         | 0.0011 | 0.9989         | 0.0012 | 0.9989         | 0.0012 | 0.6159         | 0.0005 |
|         | EW | 0.9858         | 0.4442 | 0.9727         | 0.6149 | 0.9746         | 0.5934 | -              | -      |
| 2 Layer | EH | 0.9998         | 0.0023 | 0.9998         | 0.0025 | 0.9998         | 0.0025 | 0.7105         | 0.0007 |
|         | EW | 0.9989         | 0.9946 | 0.9977         | 1.4214 | 0.9980         | 1.3325 | -              | -      |
| 3 Layer | EH | 0.9987         | 0.0025 | 0.9985         | 0.0026 | 0.9986         | 0.0025 | 0.9084         | 0.0004 |
|         | EW | 0.9850         | 0.3425 | 0.9828         | 0.3665 | 0.9832         | 0.3614 | -              | -      |

We developed predictive models using CatBoost, XGBoost, LightGBM, and Gurobi Optimizer to estimate EH across the datasets, including GSG, SGSG, SGSGS, double-layer, and triple-layer configurations. As shown in Table. III, the machine learning models exhibit high accuracy in predicting EH, with CatBoost yielding the best performance.

Key findings include:

- **CatBoost Performance:**  
Achieved an  $R^2$  of 0.9459 for EH prediction in the GSG dataset, outperforming other models.
- **Model Comparison:**  
Across datasets, CatBoost consistently outperformed XGBoost and LightGBM, while Gurobi's formula-based approach yielded lower accuracy, particularly for multi-layer structures (Table. II).

To reduce overfitting, the dataset was split into 70% training, 15% validation, and 15% testing subsets, with the split performed at the configuration level so that no identical parameter combinations appear in more than one subset.

These results confirm that machine learning techniques can accurately predict SI metrics, offering a quick evaluation method for IC design.

#### C. Evaluating SI in Different Routing Stages

We evaluated SI performance in global and detailed routing stages by calculating each bump's mean absolute percentage error (MAPE), focusing on EH and EW metrics. As an initial step, we compared global routing with detailed routing, which refines path and design rule constraints.

$$\text{MAPE}(y, \hat{y}) = \frac{1}{N} \sum_{i=0}^{N-1} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\%$$

Detailed routing improves SI metrics by reducing deviations in EH and EW, leading to better signal quality. It was particularly evident in the fully shielded configurations, where the MAPE difference between global and detailed routing highlighted the value of our method in minimizing routing errors and improving design efficiency.

**TABLE IV: Evaluating Eye Height(V) across Routing Stages**

|       | Initial Stage (Global Routing) |       |       | Final Stage (Detailed Routing) |       |       | MAPE Variations |
|-------|--------------------------------|-------|-------|--------------------------------|-------|-------|-----------------|
|       | Average                        | Worst | Best  | Average                        | Worst | Best  |                 |
| Case1 | 0.816                          | 0.792 | 0.818 | 0.824                          | 0.799 | 0.848 | 1.045%          |
| Case2 | 0.792                          | 0.782 | 0.801 | 0.806                          | 0.791 | 0.836 | 1.678%          |
| Case3 | 0.839                          | 0.833 | 0.845 | 0.845                          | 0.834 | 0.849 | 0.694%          |
| Case4 | 0.824                          | 0.818 | 0.826 | 0.829                          | 0.819 | 0.850 | 0.749%          |

**TABLE V: Evaluating Eye Width(ps) across Routing Stages**

|       | Initial Stage (Global Routing) |       |       | Final Stage (Detailed Routing) |       |       | MAPE Variations |
|-------|--------------------------------|-------|-------|--------------------------------|-------|-------|-----------------|
|       | Average                        | Worst | Best  | Average                        | Worst | Best  |                 |
| Case1 | 150.66                         | 149.2 | 150.8 | 152.01                         | 149.8 | 155.3 | 1.117%          |
| Case2 | 149.24                         | 149.0 | 149.6 | 149.54                         | 147.1 | 153.9 | 0.967%          |
| Case3 | 152.83                         | 152.0 | 154.2 | 152.95                         | 150.8 | 154.9 | 0.479%          |
| Case4 | 151.17                         | 150.8 | 151.3 | 152.32                         | 150.4 | 154.0 | 0.940%          |

## V. CONCLUSIONS

We have developed two machine learning-driven frameworks to characterize signal integrity (SI) in the InFO package. The first framework is an optimization-based model using Gurobi Optimizer, which derives a formula to predict eye height and provides a mathematical approach for signal integrity evaluation. The second framework is a CatBoost-based predictive model, which achieves higher accuracy ( $R^2 = 0.94-0.99$ ) in predicting eye height and eye width by training a predictive model using CatBoost.

Even when the ML models were trained on datasets generated from continuous-variable without integer constraints, the prediction accuracy remained high ( $R^2$  above 0.94), indicating that the strong performance is due to the chosen feature set and modeling capability rather than integer rounding effects.

The angle information enhances the model's accuracy and develops a comprehensive framework for estimating signal integrity, considering various physical features such as wire length, spacing, and multi-layer structures. Our models showed reliable performance on simple single-layer structures and complex multi-layer layouts, allowing fast signal quality evaluation and early detection of possible defects in data transmission.

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